§29. Entropy Production and Inward Heat Pinch of Plasma

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Recently, the heat was found to flow to the direction of the higher temperature, in D III-D tokamak [1]. This is in contrast to the daily experience that the heat flows from the higher temperature region to the lower temperature region. We study the rate of entropy production associated with the inward energy flow. The case is found that the entropy production rate have optimum where the energy flows in the direction of the higher temperature.

We study the simple case of plasma cylinder. The electron heating takes place at $r = r_1$, and no energy is externally given to the plasma in the region $0 < r < r_1$. $T_e(r_1)$ and $T_i(r_1)$ are ginen as boundary condition. The density profile is taken to be uniform in the region $0 < r < r_1$. The rotation effect is neglected. We study the stationary state of the core region inside of $r = r_1$, i.e., $0 < r < r_1$. The radial energy flow of electrons, q_e, that of ions, q_i, and the power transfer from the electrons to the ions, P_{ei}, are considered. The relation $q_e = -q_i$ indicates that there is no power supply from outside in the region of $0 < r < r_1$.

The entropy production rate is calculated in the region of our interest, $0 < r < r_1$. The local entropy production rate, S, is defined as $S_e = -\sum_j X^e_j F^e_j$, $S_i = -\sum_j X^i_j F^i_j$, where e and i denotes electrons and ions, X is the thermo-dynamical force, and F is the thermodynamical flux. We employ the form of X's and F's as $X_2 = \nabla T/T$, $X_3 = 1/T$, $F_2 = q_r/T$, $F_3 = -P + q_r X_2$, where P is the absorbed power and q_r is the radial heat flux. The force and flow of j=1 component is associated with the particle flux, and they do not contribute in the situation here. The global entropy production rate, $\sigma_{e,i}$, is calculated by integrating $S_{e,i}$ over the volume as $\sigma_{e,i} = \int_0^{r_1} rdr\, S_{e,i}(r)$. Integrating by part once, we have

$$\sigma_e = -\frac{2}{T_e(r_1)}\int_0^{r_1} rdr\, P_{ei} + \int_0^{r_1} rdr \frac{P_{ei}}{T_e}$$

$$\sigma_i = \frac{2}{T_i(r_1)}\int_0^{r_1} rdr\, P_{ei} - \int_0^{r_1} rdr \frac{P_{ei}}{T_i}$$

The total entropy production rate is given as $\sigma = \sigma_e + \sigma_i$.

By performing the functional derivative of σ, the temperature profiles which optimize σ are obtained. As an example, we choose the Coulomb collision process, which gives the lower boundary for the energy exchange and is employed in cases such as D III-D plasmas. We then have the relation $P_{ei} = C\,(T_e - T_i)\,T_e^{-3/2}$, where C is a numerical coefficient which is considered to be constant in this analysis. By this assumption, σ is a functional of the temperature profiles of electrons and ions.

The variation of σ with the temperature profiles is studied. The test function for the temperature profile is chosen as

$$T_{e,i}(r) = T_{e,i}(0) - [T_{e,i}(0) - T_{e,i}(r_1)]\left(\frac{r}{r_1}\right)^2 .$$

The variation of the temperature profile is characterized by the central temperature $T_{e,i}(0)$. By this simplification, the functional derivative is replaced by the partial derivative with respect to $T_{e,i}(0)$. Figure 1 shows the values of $\{T_e(0)^*, T_i(0)^*\}$, which maximize σ, for various values of $T_e(r_1)/T_i(r_1)$. Appreciable difference between $T_e(0)$ and $T_i(0)$ is necessary to sustain this system. The negative entropy production rate associated with the heat pinch is compensated by the excess entropy production of colder ions.

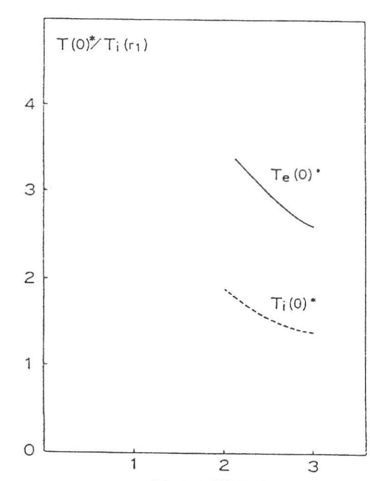

Fig. 1 Central temperature which optimizes the entropy production rate as a function of the ratio $T_e(r_1)/T_i(r_1)$.

References
1) T. C. Luce, C. C. Petty and J. C. M. Dehaas: Phys. Rev. Lett. **68** (1992) 52.
2) S.-I. Itoh and K. Itoh: Research Report FURKU Report 94-01 (08).